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SOLAR-WIND WAVES AND THEIR STABILITY IN THE FRAMEWORK OF THE HALL MAGNETOHYDRODYNAMICS

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Abstract: *It is well established now that the solar atmosphere, from photosphere to the corona and the solar wind is a highly structured medium. Satellite observations have confirmed the presence of steady flows there. Here, we investigate the propagation of magnetohydrodinamic (MHD) surface waves travelling along an ideal incompressible flowing plasma cylinder (flux tube) surrounded by flowing plasma environment in the framework of the Hall magnetohydrodynamics. The waves' propagation characteristics are studied in a reference frame moving with the mass flow outside the tube. In general, flows change waves' phase velocities compared to their magnitudes in a static MHD flux tube and the Hall effect extends the number of the possible wave dispersion curves. It turns out that while the kink waves in the framework of the standard magnetohydrodynamics are unstable against the Kelvin–Helmholtz instability, they become stable when the Hall effect is taken into account. The sausage waves are stable in both considerations.*

ВЪЛНИ В СЛЪНЧЕВИЯ ВЯТЪР И ТЯХНАТА УСТОЙЧИВОСТ В РАМКИТЕ НА ХОЛОВАТА МАГНИТОХИДРОДИНАМИКА

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Ключови думи: *магнитохидродинамични вълни, неустойчивост на Келвин–Хелмхолц, плазма на слънчевия вятър*

Резюме: *Установено е, че слънчевата атмосфера, от фотосферата до короната и слънчевия вятър, е силно структурирана среда. Спътникови наблюдения потвърдиха наличието на стационарни потоци в нея. Тук изследваме разпространението на магнитохидродинамични (МХД) повърхнинни вълни, разпространяващи се в идеален несвиваем течащ плазмен цилиндър (магнитна тръба), обграден от течаща плазмена среда, в рамките на Холовата магнитохидродинамика. Дисперсионните характеристики на вълните са изучавани в отправна система, движеща се с масовия поток извън тръбата. Потоците променят фазовите скорости на вълните в сравнение с техните големини в неподвижна МХД магнитна тръба, а ефектът на Хол увеличава броя на възможните дисперсионни криви на вълните. Оказва се, че докато kink-вълните, в рамките на стандартната магнитохидродинамика, са неустойчиви и неустойчивостта е от вида на Келвин–Хелмхолц, те стават устойчиви, когато се отчита влиянието на ефекта на Хол. Другите характерни МХД вълни – sausage-вълните – са устойчиви в рамките на двете описания.*

Introduction

Various waves and oscillations which occur in structured solar atmosphere were intensively studied over the past three decades [1]. Next step in investigating the wave phenomena in solar and stellar atmospheres was the consideration of steady flows there. Satellite measurements performed by *SOHO*, *Ulysses*, *Yohkoh*, *Wind*, *ACE*, and more recently by *STEREO*, of plasma characteristics of, for instance, the solar wind and coronal plumes flows, such as the magnetic field, the thermal and flow velocity and density of plasma or plasma compositions, are important to understand the various plasma wave modes which may arise. However, wave analysis requires further information and special tools to identify which set of modes is contributing to observed wave features. In practice, one may use filters to perform the so-called *pattern recognition* to detect the various kind of modes that may propagate in plasma and to determine their contribution to the wave energy [2]. Another important issue is the question for the waves' stability. The magnetosonic waves in structured atmospheres with steady flows have been examined by Nakariakov and Roberts [3], Nakariakov *et al*. [4], Andries and Goossens [5]. Andries and Goossens studied also the conditions at which resonant flow and Kelvin–Helmholtz instability take place.

It is worth pointing out that all the aforementioned studies were performed in the framework of the standard magnetohydrodynamics. It was Lighthill [6] who pointed out 50 years ago that for an adequate description of wave phenomena in fusion and astrophysical plasmas one has to include the Hall term, $m_i(\mathbf{j} \times \mathbf{B})/e\rho$, in the generalized Ohm's law. That approach is termed Hall magnetohydrodynamics (Hall MHD). In this way, it is possible to describe waves with frequencies up to $\omega \approx \omega_{ci}$. Because the model still neglects the electron mass, it is limited to frequencies well below the lower hybrid frequency: $\omega << \omega_{lh}$. Generally speaking, the theory of Hall MHD is relevant to plasma dynamics occurring on length scales shorter than an ion inertial length, $L < l_{Hall} = c/\omega_{pi}$ (where c is the speed of light and ω_{pi} is the ion plasma frequency), and time scales of the order or shorter than the ion cyclotron period ($t < 1/\omega_{ci}$) [7]. Thus the Hall MHD should affect the dispersion characteristics of the MHD waves in spatially bounded magnetized plasmas. An extensive review for the studies of waves' propagation in bounded MHD plasmas in the framework of both the standard and Hall magnetohydrodynamics the reader can find in Ref. [8].

Here, we investigate the influence of flow velocities on the dispersion characteristics and stability of hydromagnetic surface waves (sausage and kink modes) travelling along an infinitely conducting, magnetized jet moving past also (with a different speed) infinitely conducting, magnetized plasma. If in the solar corona plasma β (the ratio of gas to magnetic pressure) is much less than unity, in the solar-wind fluxes tubes it is $\beta \approx 1$. Since we are going to study the wave propagation in flowing solar-wind plasma, we can assume that we have a 'high-β' magnetized plasma and treat it as an incompressible fluid.

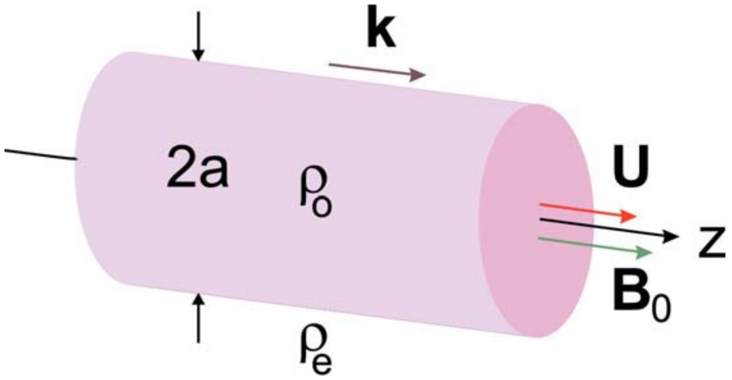

Fig. 1: Geometry of a solar wind flux tube containing flowing plasma.

For simplicity, we consider a cylindrical jet of radius a (immersed together with the environment in a constant magnetic field $\mathbf{B}_0$ directed along the z-axis), allowing for different plasma densities within and outside the jet, ρ_o and ρ_e, respectively (Fig. 1). The most natural discontinuity which occurs at the surface bounding the cylinder is the tangential one because it is the discontinuity that ensures a static pressure balance. For typical values of the background constant magnetic field $B_0 = 5 \times 10^{-9}$ T and the electron number density inside the jet $n_o = 2.43 \times 10^6$ m^{-3} at 1 AU, the ion cyclotron frequency $\omega_{ci}/2\pi$ = 76 mHz, the Alfvén speed v_{Ao} = 70 km/s, and the Hall scale length (= v_{Ao}/ω_{ci}, which is equivalent to c/ω_{pi}) is $l_{Hall} \approx 150$ km. This scale length is small, but not negligible compared to tube's radius of a few hundred kilometers. Here, we introduce a scale parameter $\varepsilon = l_{Hall}/a$ called the *Hall parameter*. In the limit of $\varepsilon \to 0$, the Hall-MHD system reduces to the conventional MHD system. Our choice for that parameter is $\varepsilon = 0.4$. The flow speeds of the jet and its environment are generally rather irregular. For investigating the stability of the travelling MHD waves it is convenient to consider the wave propagation in a frame of reference attached to the flowing environment. Thus we can define the relative flow velocity $\mathbf{U}^{rel} = \mathbf{U}_o - \mathbf{U}_e$ (U_o and U_e being the steady flow speeds correspondingly inside and outside the flux tube) as an entry parameter whose value determines the stability/instability status of the jet. As usual, we normalize that relative flow velocity with respect to the Alfvén speed in the jet, $v_{Ao} = B_0/(\mu_0\rho_o)^{1/2}$, and call it Alfvénic Mach number M_A, omitting for simplicity the superscript "rel". Another important entry parameter of the problem is $\eta = \rho_e/\rho_o$. It turns out that the waves' dispersion characteristics and their stability critically depend on the magnitude of η. In this study, we take η = 0.568 which, for a compressible plasma approach, would correspond to $v_{Ao} = c_{so}$ = 70 km/s and v_{Ae} = 100 km/s, c_{se} = 70 km/s, respectively, where c_s is the sound speed in the corresponding medium. Thus the waves' dispersion characteristics (the dependence of the wave phase velocity $v_{ph} = \omega/k_z$ on the wave number k_z) and their stability states are determined by the three parameters η, ε, and M_A, two of which are fixed (η and ε) and the third one, M_A, is running.

Basic equations and dispersion relations

As seen in Fig.1, the three important vectors, the embedded magnetic field $\mathbf{B}_0$, the relative flow velocity $\mathbf{U}$, and the wave vector $\mathbf{k}$, lie along the z-axis. The basic equations which govern the propagation of Hall-MHD waves in flowing incompressible plasma are the equations for the perturbed fluid velocity $\mathbf{v}_1$ and perturbed wave magnetic field $\mathbf{B}_1$:

(1) $\rho\partial\mathbf{v}_1/\partial t + \rho(\mathbf{U}\cdot\nabla)\mathbf{v}_1 + \nabla(\mathbf{B}_0\cdot\mathbf{B}_1)/\mu_0 + (\mathbf{B}_0\cdot\nabla)\mathbf{B}_1/\mu_0 = 0,$

(2) $\partial\mathbf{B}_1/\partial t + (\mathbf{U}\cdot\nabla)\mathbf{B}_1 - (\mathbf{B}_0\cdot\nabla)\mathbf{v}_1 + \mathbf{B}_0\nabla\cdot\mathbf{v}_1 + v_A^2(\mathbf{z}\cdot\nabla)\nabla\times\mathbf{B}_1/\omega_{ci} = 0,$

with the constrains

(3) $\nabla\cdot\mathbf{v}_1 = 0$ and $\nabla\cdot\mathbf{B}_1 = 0,$

where $\mathbf{z}$ is the unit vector of the z-axis; the other notation is standard. After Fourier transforming the perturbed quantities proportional to $g(r)\exp(-\mathrm{i}\omega t + \mathrm{i}m\varphi + \mathrm{i}k_z z)$, we get a second order differential equation for the perturbed magnetic pressure $p_{1\mathrm{mag}}$:

(4) $[\mathrm{d}^2/\mathrm{d}r^2 + (1/r)\mathrm{d}/\mathrm{d}r - (k_z^2 + m^2/r^2)]B_0B_{1z}/\mu_0 = 0,$

as well as an expression for the perturbed radial fluid velocity component in terms of the first derivative of $p_{1\mathrm{mag}}$

(5) $v_{1r} = -\mathrm{i}\,\mu_0 F(\mathrm{d}p_{1\mathrm{mag}}/\mathrm{d}r)/(B_0k_z)^2,$

where

$$F = [(\omega - \mathbf{k}\cdot\mathbf{U})(C-1)]/[(C-1)^2 - \sigma^2]$$

with

$$C = [(\omega - \mathbf{k}\cdot\mathbf{U})/(k_z v_A)]^2 \quad \text{and} \quad \sigma = (\omega - \mathbf{k}\cdot\mathbf{U})/\omega_{ci}.$$

It is worth noticing that we have different Cs and σs inside and outside the jet.

The solutions to the differential equation (5) are the modified Bessel functions, more specifically

$$p_{1\mathrm{mag}}(\mathrm{r}) = AI_m(k_z r) \quad \text{for} \quad r < a$$

and

$$p_{1\mathrm{mag}}(\mathrm{r}) = BK_m(k_z r) \quad \text{for} \quad r > a.$$

Accordingly, the expressions for v_{1r} inside and outside the jet are:

$$v_{1r}(\mathrm{r} < \mathrm{a}) = -\mathrm{i}\mu_0 F_o A I_m'(k_z r)/(B_0k_z)^2$$

and

$$v_{1r}(\mathrm{r} > \mathrm{a}) = -\mathrm{i}\mu_0 F_e B K_m'(k_z r)/(B_0k_z)^2,$$

where

$$F_o = [(\omega - \mathbf{k}\cdot\mathbf{U})(C_o - 1)]/[(C_o - 1)^2 - \sigma_o^2] \quad \text{and} \quad F_e = \omega(C_e - 1)/[(C_e - 1)^2 - \sigma_e^2]$$

with

$$C_o = [(\omega - \mathbf{k}\cdot\mathbf{U})/(k_z v_{Ao})]^2,\quad C_e = \omega^2/(k_z v_{Ae})^2,\quad \sigma_o = (\omega - \mathbf{k}\cdot\mathbf{U})/\omega_{ci},\quad \text{and} \quad \sigma_e = \omega/\omega_{ci}.$$

For solving our problem we need two boundary conditions, applied at the $r = a$ interface. These boundary conditions are [9]:

- continuity of the perturbed interface $v_{1r}/(\omega - \mathbf{k}\cdot\mathbf{U})$,
- continuity of the perturbed magnetic pressure $p_{1\mathrm{mag}}$.

The application of the boundary conditions yields the following dispersion relations for *sausage* (m = 0) and *kink* (m = 1) modes running along the jet's interface:

(6) $G_o I_m'(k_z a)/I_m(k_z a) - G_e K_m'(k_z a)/K_m(k_z a) = 0,$

where

$$G_o = (C_o - 1)/[(C_o - 1)^2 - \sigma_o^2] \quad \text{and} \quad G_e = (C_e - 1)/[(C_e - 1)^2 - \sigma_e^2].$$

As seen, the wave frequency ω is Doppler-shifted inside the jet, and also the two modes are pure surface waves. If we ignore the Hall term ($\sigma_{o,e} = 0$), the above equation reduces to [3]:

(7) $\eta(\omega^2 - k_z^2 v_{Ae}^2) I_m'(k_z a)/I_m(k_z a) - [(\omega - \mathbf{k} \cdot \mathbf{U})^2 - k_z^2 v_{Ao}^2] K_m'(k_z a)/K_m(k_z a) = 0,$

i.e., we recover well-known dispersion relations.

As we are interested in the stability of the surface waves travelling along the jet interface, we have to assume that the wave frequency is complex, i.e., $\omega \to \omega + i\gamma$, where γ is the expected instability growth rate. Since we will plot dispersion diagrams as dependencies of the wave phase velocity v_{ph} on the wave number k_z, we normalize all quantities by defining the dimensionless wave phase velocity $V_{ph} = \omega/k_z v_{Ao}$, wave number $K = k_z a$, and the relative Alfvénic Mach number $M_A = U/v_{Ao}$ to get the dimensionless form of our dispersion relations. It is worth investigating in parallel both waves' dispersion relations (those in the framework of the conventional magnetohydrodynamics and the Hall-MHD waves) in order to see how the Hall term modifies waves' dispersion curves and the instability growth rates when the waves become unstable. Thus the dimensionless forms of the aforementioned dispersion relations are:

(8) $[(V_{ph} - M_A)^2 - 1] Z_2 I_m'(K)/I_m(K) - (\eta V_{ph}^2 - 1) Z_1 K_m'(K)/K_m(K) = 0,$

where

$$Z_1 = [(V_{ph} - M_A)^2 - 1]^2 - \varepsilon^2 K^2 (V_{ph} - M_A)^2,$$

$$Z_2 = (\eta V_{ph}^2 - 1)^2 - \varepsilon^2 K^2 V_{ph}^2.$$

In above expressions $\varepsilon = l_{Hall}/a$ is the Hall parameter. The dispersion relation of the conventional surface MHD waves takes the form:

(9) $(\eta V_{ph}^2 - 1)\, I_m'(K)/I_m(K) - [(V_{ph} - M_A)^2 - 1]\, K_m'(K)/K_m(K) = 0.$

It is easy to see that the dispersion relation (9) of the conventional MHD surface waves is a quadratic one and its roots are:

$$V_{ph} = (-M_A B \pm D^{1/2})/(\eta A - B),$$

where

$$A = I_m'(K)/I_m(K), \qquad B = K_m'(K)/K_m(K),$$

and the discriminant D is

$$D = M_A^2 B^2 + (\eta A - B)[(1 - M_A^2)B - A].$$

Obviously, if D is non-negative (i.e., greater than or equal to zero), then

$$\mathrm{Re}V_{ph} = (-M_A B \pm D^{1/2})/(\eta A - B), \qquad \mathrm{Im}V_{ph} = 0,$$

else

$$\mathrm{Re}V_{ph} = -M_A B/(\eta A - B), \qquad \mathrm{Im}V_{ph} = D^{1/2}/(\eta A - B).$$

Numerical results and discussion

While the dispersion equation (9) of standard MHD surface waves is a quadratic one and its roots can be expressed in closed forms, that of the Hall-MHD waves, Eq. (8), is a complex polynomial of sixth order and it can be solved only numerically. We have used the Müller method for finding the complex roots of that equation. Before starting the numerical procedure for solving either dispersion equation, we have to specify the jet's entry parameters, notably η, M_A, and the Hall parameter ε for the Hall waves. Since the dispersion relations are sensitive to the values of η, we have calculated the waves' dispersion curves (and growth rates when the waves are unstable) for a value of that parameter which corresponds to the jet parameters listed in the Introduction section of this paper, namely $\eta = 0.586$. The relative Alfvénic Mach number M_A is a running entry parameter whose values vary from zero to some reasonable numbers. Recall that the value of the Hall parameter is $\varepsilon = 0.4$. We will first display the results for the kink mode, and later on for the sausage one.

It is clear from the explicit solutions to the dispersion equation of the conventional kink waves that for each M_A we have two curves when the waves are stable and *only* one curve as they become unstable. That is seen in Figs. 2 and 3. For relatively small Alfvénic Mach numbers one observes one forward and one backward propagating wave (look at Fig. 2). For a little bit bigger M_As, say $M_A > 1.5$, both waves become forward running ones and at $M_A = 2.35$ (Fig. 3) they start to merge forming close dispersion curves. At some critical value of M_A, in our case at $M_A = 2.42$, the waves become unstable

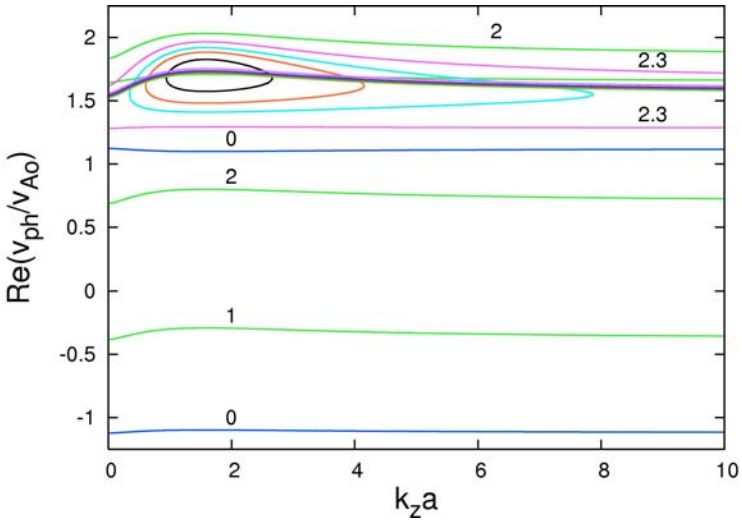

Fig. 2: Dispersion curves of forward and backward propagating conventional kink waves for different values of the relative Alfvénic Mach number M_A.

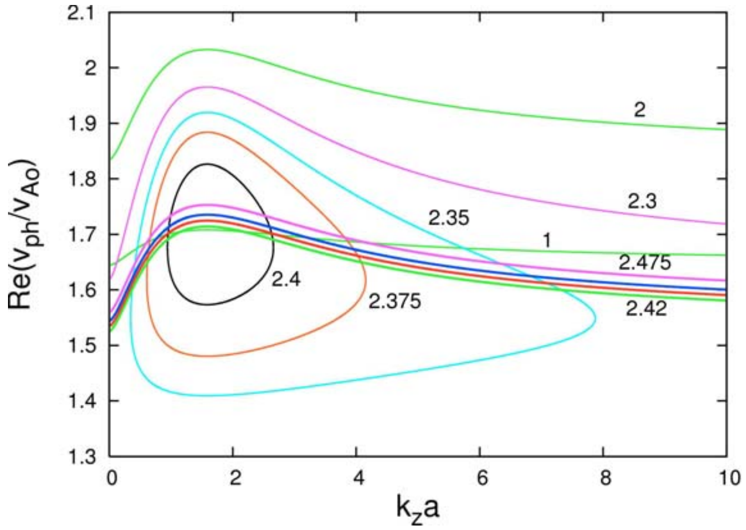

Fig. 3: Zoomed dispersion curves of forward propagating kink waves shown in Fig. 2.

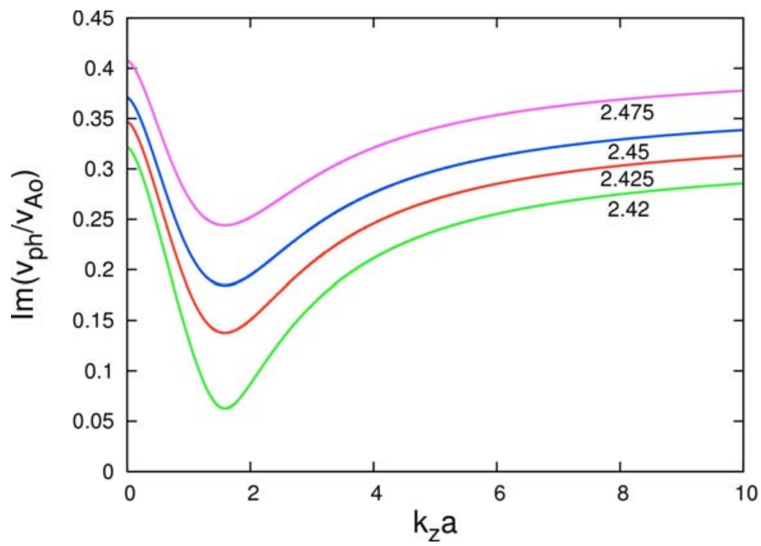

Fig. 4: Growth rates of the unstable conventional kink waves.

and their growth rate can be seen in Fig. 4. The found instability is of the Kelvin–Helmholtz type and the growth rates for other three relative Alfvénic Mach numbers are plotted in the same figure – the corresponding dispersion curves are clearly depicted in Fig. 3. We have to emphasise that according to Andries and Goossens [5] the Kelvin–Helmholtz instability onset starts at $U > v_{Ao} + v_{Ae}$, or equivalently at

$$M_A > 1 + 1/\eta^{1/2}.$$

In our case (η = 0.586) above inequality requires $M_A > 2.31$ in order to expect an instability onset. Obviously, our numerical calculations confirm the applicability of that criterion.

One must mention that one can get dispersion curves and growth rates of kink unstable waves for negative values of the relative Alfvénic Mach number, however, those waves are backward propagating ones and not acceptable from a physical point of view. The reason for that conclusion is that the MHD surface waves are the incompressible vestige of the slow magnetosonic waves of the compressible magnetohydrodynamics which propagate with group velocity, $\mathbf{v}_A$, either parallel or anti-parallel to $\mathbf{B}_0$ depending on the sign of k_z [8].

As we have already mentioned, the dispersion relations of the Hall-MHD surface waves are polynomials of sixth order possessing all non-zero coefficients. That means we have to expect to get six different dispersion curves for each M_A. In Fig. 5 we show the dispersion curves of the kink waves

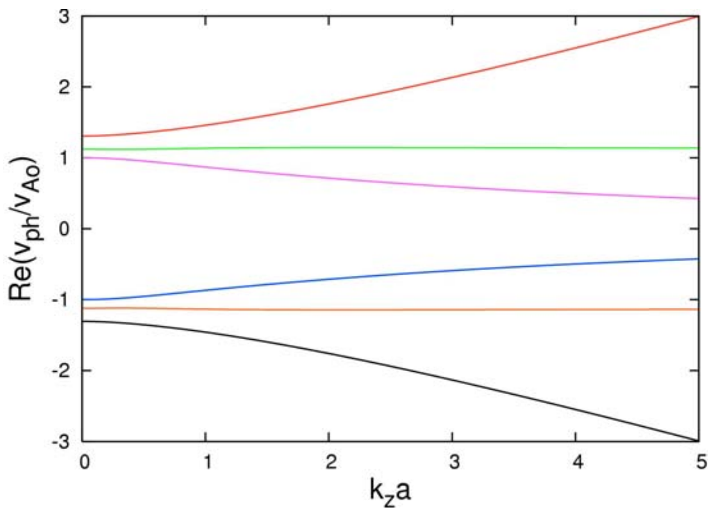

Fig. 5: Dispersion curves of forward and backward propagating Hall-MHD kink waves for M_A = 0.

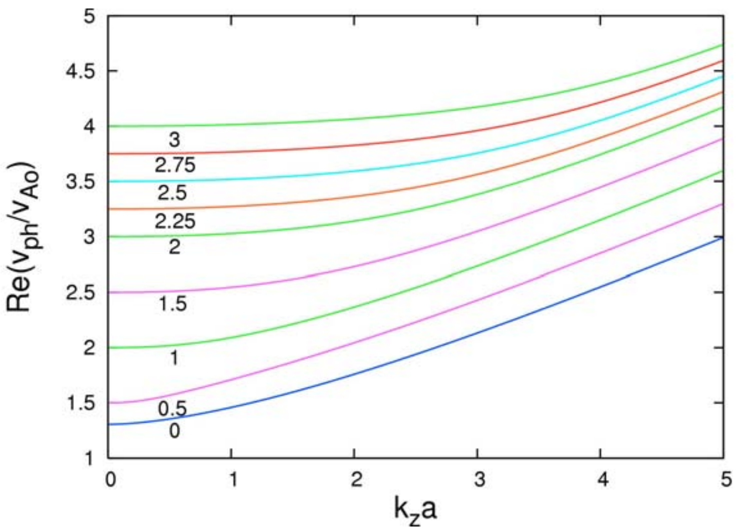

Fig. 6: Dispersion curves of the fast forward propagating Hall-MHD kink waves for different values of M_A.

propagating along a static (M_A = 0) flux tube. It is seen that we have three forward and three backward waves. Considering only the forward waves one can divide them into three categories, namely fast (super-Alfvénic) kink waves, Alfvénic kink waves, and slow (sub-Alfvénic) kink waves. Figure 6 shows the evolution of the fast forward kink waves with increasing the relative Alfvénic Mach number M_A. It turns out that all the dispersion curves correspond to stable wave propagation. The evolution of the Alfvénic kink waves can be seen in Fig. 7. It is evident that in a flowing plasma their normalized

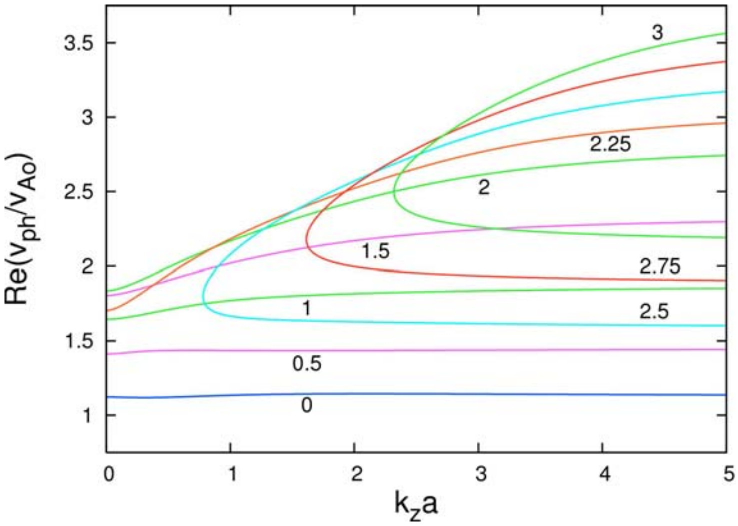

Fig. 7: Dispersion curves of the Alfvénic forward propagating Hall-MHD kink waves for different values of M_A.

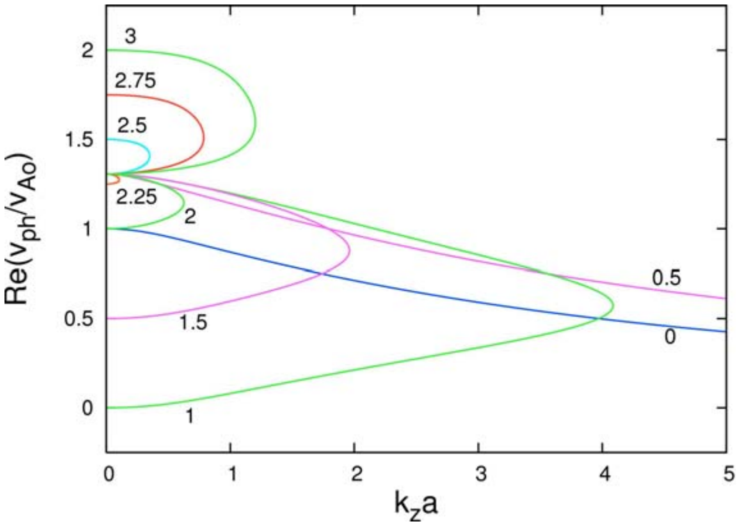

Fig. 8: Dispersion curves of the sub-Alfvénic forward propagating Hall-MHD kink waves for different values of M_A.

phase velocities increase and the waves actually become super-Alfvénic ones. Moreover for greater values of M_A (starting from 2.5) the form of their dispersion curves drastically changes – for small normalized wave numbers $k_z a$ there exist regions of non-propagation and each dispersion curve consists of two branches that merge at a specific $k_z a$ – for example, with M_A = 2.5 that value of $k_z a$ is 0.78. All dispersion curves plotted in Fig. 7 correspond to stable wave propagation. We observe a similar picture for the initially slow (sub-Alfvénic) forward Hall-MHD kink waves whose dispersion curves are shown in Fig. 8. What is more interesting, with the increasing the relative Alfvénic Mach number M_A the waves generally become super-Alfvénic (with the exception of low-branch curves corresponding to M_A equal to 1 and 1.5) and for some values of M_A (for instance at M_A = 2.25) the region of wave propagation is extremely narrow. The most surprising result is that for relative Alfvénic Mach numbers in the region of 2.4–2.6 where one may expect the onset of the Kelvin–Helmholtz instability (like in the case of the conventional MHD kink waves) the waves are still stable! The Müller method which has been used for solving the complex dispersion equations does not yield complex roots – the imaginary parts of the normalized wave phase velocities always were equal to zero. In other words, it turns out that the Hall term in the generalized Ohm's law not only changes the form and numbers of the dispersion curves but also stabilizes the wave propagation along the jet against the Kelvin–Helmholtz instability.

The dispersion curves of the conventional sausage MHD surface waves travelling along the jet have been calculated by using Eq. (9) with $m = 0$ and are displayed in Figs. 9 and 10. For fairly small

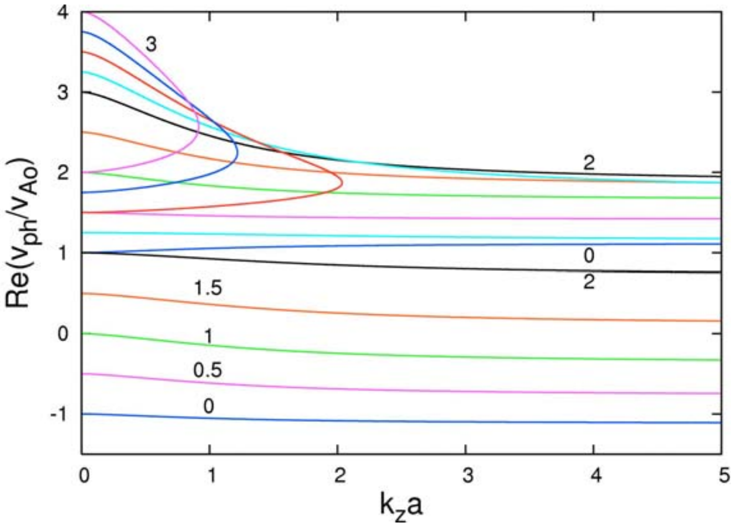

Fig. 9: Dispersion curves of forward and backward propagating conventional sausage waves for different values of the relative Alfvénic Mach number M_A.

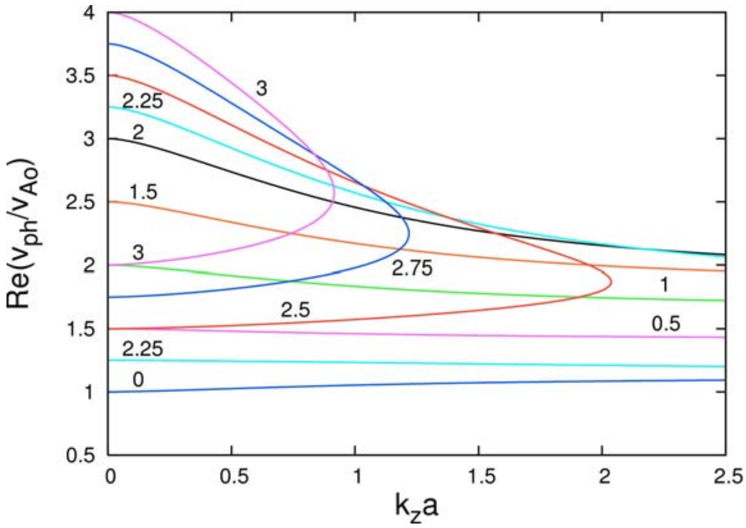

Fig. 10: Zoomed dispersion curves of forward propagating sausage waves shown in Fig. 9.

relative Alfvénic Mach numbers, less than or equal to 1, we have both forward and backward propagating waves. With the increasing of M_A the shape of the dispersion curves changes and at M_A = 2.5 both curves merge (look at Fig. 10) narrowing the propagation's range of the waves. Moreover, for such big enough relative Alfvénic Mach numbers for a fixed normalized wave number $k_z a$ we have multiple (in our case two) solutions for the normalized wave's phase velocity. Which one of these two velocities will be registered during the wave propagation cannot be predicted by the theory. It is rather astonishing that we get *no* complex solutions to the wave dispersion equation, i.e., the conventional sausage MHD surface waves are stable against the Kelvin–Helmholtz instability.

Let us see whether the inclusion of the Hall effect will change the stability state of the sausage mode. In the next four figures we show the dispersion curves of the Hall-MHD sausage surface waves running on a solar-wind jet. As in the case of the kink waves, for M_A = 0 (propagation along a static flux tube) we obtain six distinct dispersion curves for the Hall-MHD sausage waves – three of them correspond to forward wave propagation and the next three to backward wave propagation. When considering only the forward propagating modes we recognize, like before, three different types of waves (see Fig. 11): one fast (super-Alfvénic) wave, one practically Alfvénic wave, and one slow (sub-

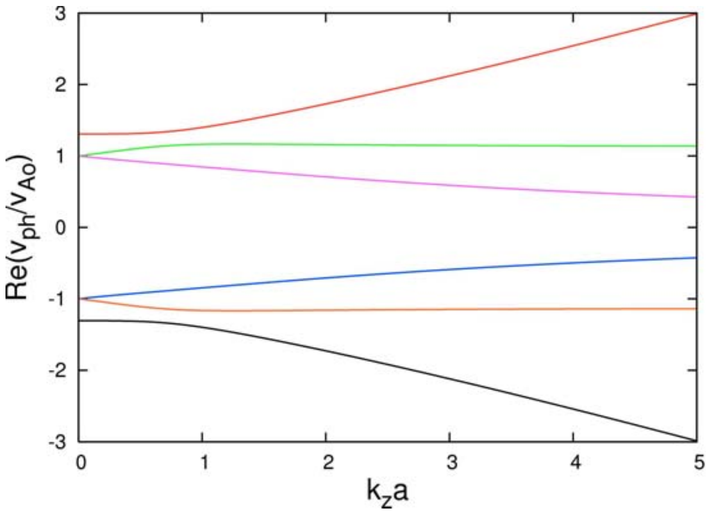

Fig. 11: Dispersion curves of the fast forward propagating Hall-MHD sausage waves for different values of M_A.

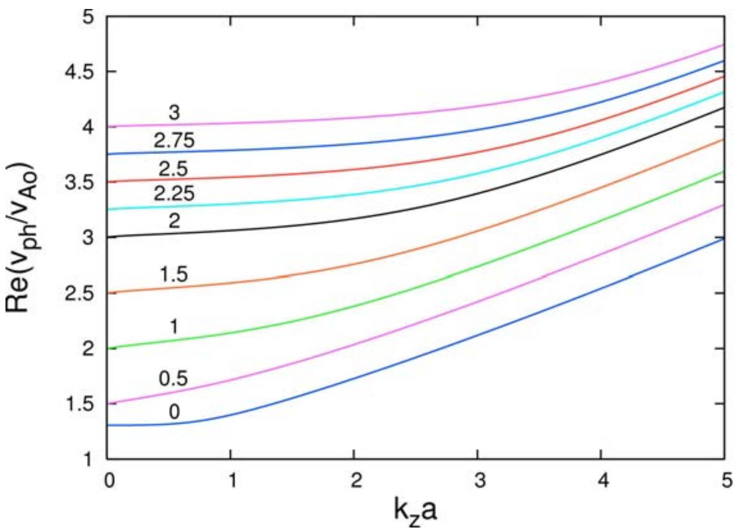

Fig. 12: Dispersion curves of the fast forward propagating Hall-MHD kink waves for different values of M_A.

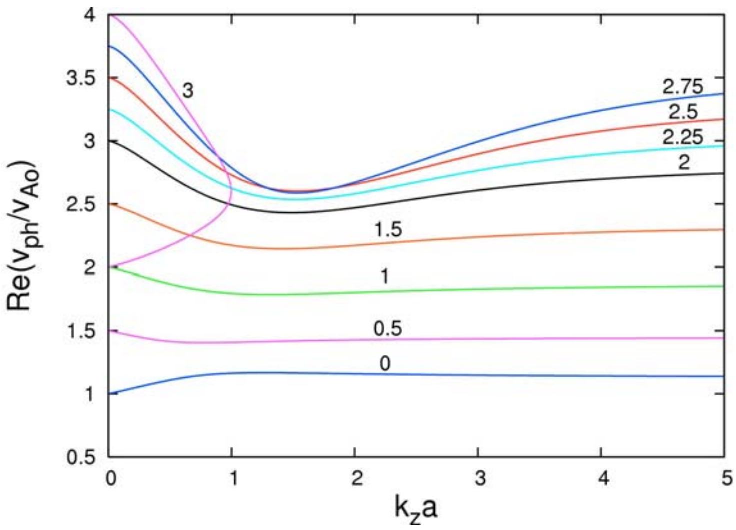

Fig. 13: Dispersion curves of the Alfvénic forward propagating Hall-MHD sausage waves for different values of M_A.

Alfvénic) wave. While the evolution of the fast Hall-MHD sausage waves with the inclusion of flow is similar to that of the kink waves (compare Figs. 6 and 12), the evolution of the initially almost Alfvénic sausage waves is completely different (compare Figs. 7 and 13). It is seen in Fig. 13 that those waves quickly become super-Alfvénic and only at great enough relative Alfvénic Mach numbers their propagation's ranges are severely reduced – look, for example, at the curve labeled by 3 in Fig. 13. Figure 14 shows the dispersion curves of Hall-MHD sausage waves for various relative Alfvénic Mach

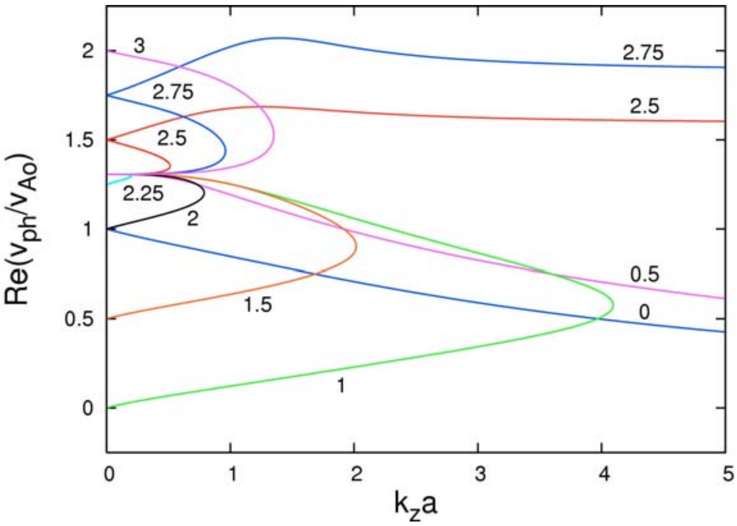

Fig. 14: Dispersion curves of the sub-Alfvénic forward propagating Hall-MHD sausage waves for different values of M_A.

numbers. It is seen, like in the case of the kink Hall-MHD waves, that the sausage waves possess semi-closed dispersion curves which significantly narrow the propagation's range of the waves – look, for instance, at the curve labelled by 2.25 in Fig. 14. Here, however, we observe a notable peculiarity: for M_A = 2.5 and 2.75 in the same range of the normalized waves' phase velocities there appear two dispersion curves (see the curves labelled by 2.5 and 2.75 on the right side of the plot in Fig. 14), which initially (for zero or small relative Alfvénic Mach numbers) belong to backward propagating Hall-MHD sausage waves. Probably it is not too surprising now to say that the sausage Hall-MHD waves, similarly to the conventional ones, are stable against the Kelvin–Helmholtz instability. Their stability state is not changed for negative M_As, either. In that case, however, there occur fast sausage waves whose normalized phase velocities are smaller than those of the super-Alfvénic waves for positive values of the relative Alfvénic Mach numbers. Similar phenomenon takes place for the kink Hall-MHD waves, too.

As we have mentioned in the Introduction section, the dispersion characteristics of both conventional and Hall-MHD waves depend upon the value of the entry parameter η. We have performed calculations for a wide range of magnitudes of that important parameter, namely η = 0.16, 0.98, 4, 10, and, of course, for η = 0.586. The obtained dispersion diagrams really looked differently, but their general features for a given eigenmode are more or less rather similar.

Conclusion and outlook

Let us summarize the main findings of our study. In investigating the wave propagation along an incompressible plasma jet moving with respect to the environment with a constant speed U we had to take into account the influence of two factors: (i) the Hall term in the generalized Ohm's law, and (ii) the flow itself. The combining effect of these two factors can be expressed as follows:

- The Hall term generally expands the range of propagation of the wave modes not only for static tubes but also for jets with $M_A \neq 0$. While the number of dispersion curves of a conventional MHD surface mode for a fixed M_A is two (one for forward propagating waves and other for backward propagating waves), for a Hall-MHD surface mode that number is six – we have generally three forward travelling waves and three backward running ones.
- With increasing the relative Alfvénic Mach number, M_A, at some its critical value the conventional kink waves become unstable and the instability is of the Kelvin–Helmholtz type. It turns out, however, that the kink Hall-MHD waves are stable for any relative Alfvénic Mach number (be it positive or negative).

- The sausage surface waves are stable against the Kelvin–Helmholtz instability for any relative Alfvénic Mach number both in the framework of the standard magnetohydrodynamics and the Hall one.
- The Hall term in the generalized Ohm's law, except of extending the number of the possible dispersion curves, also dramatically changes the shapes of the curves corresponding in the limit $M_A \rightarrow 0$ to slow (sub-Alfvénic) and almost Alfvénic waves. In particular, for big enough M_As their ranges of propagation are narrowed and for a fixed normalized wave number $K = k_z a$ one may have two distinctly different normalized phase velocities.

It is interesting and very instructive to compare our results with those associated with the propagation of Hall-MHD waves in a flowing incompressible plasma slab along the constant external magnetic field $\mathbf{B}_0$ [8]. In the slab geometry the Hall term generally limits the range of propagation of the wave modes not only for static tubes/layers but also for jets possessing positive Alfvénic Mach numbers. The limiting normalized wave number (see Eq. (18) in Ref. [8])

$$K_{\text{limit}} = (1 + \eta)^{1/2}/\varepsilon$$

is specified by two plasma parameters: the densities ratio of the two plasma media (outside and inside the jet), η, and the Hall parameter, ε. With approaching that wave number the wave phase velocity becomes very high. When M_A is negative the real part of the phase velocity of the eigenmodes (kink and sausage waves) is forced *due to the flow's presence* to go beyond that limiting wave number in a region where the wave becomes unstable (or if you prefer, overstable). The instability which occurs is of the Kelvin–Helmholtz type. The main conclusion in Ref. [8] is that the Hall current keeps stable the surface modes travelling in flowing solar plasmas within the dimensionless wave number range between 0 and K_{limit} for each relative Alfvénic Mach number. Instability of the Kelvin-Helmholtz type for the forward propagating waves is possible only at negative Alfvénic Mach numbers in a wave number range lying beyond the K_{limit}. The instability can disappear as $|M_A|$ reaches some value depending on the magnitude of the parameter η (look at Figs. 12 and 13 and read the comments for them in Ref. [8]). One of the unexpected surprises of our study in cylindrical geometry was that we *did not get* such a K_{limit}. Hence, if we extrapolate the main conclusion of Ref. [8], we can claim that the Hall-MHD surface waves propagating along a cylindrical incompressible solar-wind plasma jet should be stable against the Kelvin–Helmholtz instability. It remains to be seen whether this statement will be valid when the plasma compressibility is taken into account. An examination of the dispersion characteristics of the Hall-MHD eigenmodes and their stability status in a compressible cylindrical solar-wind jet are in progress and will be reported elsewhere.

Acknowledgments

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